2-6-4 Scientific Notation

Scientific notation is used to write very large and very small numbers.
The distance to the sun is150, 000,000km or 1.5×10^8 km.
The bacteria Streptococcus pyogenes is 0.0000625 cm or 6.25×10^{-5} cm long.

$4 \times 10^3 = 4 \times 10 \times 10 \times 10 = 4 \times 1000 = 4000$ Notice: the exponent on the 10 is 3 and the decimal after the 4 moved to the right 3 spaces.

$2.3 \times 10^5 = 23000$ moving the decimal right 5 spaces.
2.3×10^5 is scientific notation and 23000 is decimal notation.

Practice: Write the following in decimal notation.
a) $4.6 \times 10^8 =$ $5.2 \times 10^3 =$ $3.4 \times 10^{12} =$

b) $7 \times 10^3 =$ $8 \times 10^8 =$ $1 \times 10^{10} =$

c) $1.123 \times 10^9 =$ $5.453 \times 10^4 =$ $6.5874 \times 10^3 =$

Rule: $a \times 10^m$ is scientific notation if $1 \le a < 10$ and m is an integer.

$2.3 \times 10^8 = 23 \times 10^7 = 0.23 \times 10^9 = 230 \times 10^6$ But only one of these is in scientific notation.

2.3 is between 1 and 10 so the correct form of the number in scientific notation is 2.3×10^8.
Although the other numbers are the same as 2.3x108, they are not scientific notation.
3.21×10^2 is the scientific form of 321.

To write 45600000 in scientific notation, $4.56 \times 10^7 = 45600000$
move the decimal so only one digit is in front and drop ending zeros. 4.56
Multiply by ten to the power that matches the number of spaces the decimal was moved.

Practice: Write the following in scientific notation.
a)   8,900,000,000,000=   2,000,000,000,000,000=   178,000,000,000,000=

b)   1,000,000,000,000=   24,000,000,000,000=   145,000,000,000,000,000=

c)   8,431,000,000,000,000=   1,000,000,000,000,000,000,000=   5,000,000,000,000=

Recall $10^{-4} = \dfrac{1}{10^4} = \dfrac{1}{10000}$ so $6.8 \times 10^{-4} = \dfrac{6.8}{10000} = 0.00068$ Dividing by 10000 moves the decimal 4 spaces to the left.

To change a small number in scientific notation to decimal notation, move the decimal to the left the number of spaces indicated by the power of 10.

$5.43 \times 10^{-3} = 0.00543$ (left) and $6.43 \times 10^3 = 6430$ (right)

Practice: Write the following in decimal notation. Watch the signs.
a) $4.6 \times 10^{-8} =$ $5.2 \times 10^{-3} =$ $3.4 \times 10^{-12} =$

b) $7 \times 10^{-3} =$ $1.8 \times 10^{-8} =$ $1 \times 10^{-10} =$

c) $1.123 \times 10^{-9} =$ $5.453 \times 10^{-5} =$ $6.5874 \times 10^{-3} =$

d) $4.8 \times 10^{3} =$ $2.3 \times 10^{-5} =$ $4.5 \times 10^{-8} =$

e) $3 \times 10^{-5} =$ $3.3 \times 10^{-6} =$ $8.1 \times 10^{-9} =$

To write a small decimal notation number to scientific notation, move the decimal after the first nonzero digit. Count how many spaces the decimal was moved. This is the power of 10 the number would be divided by. Make it a negative exponent.

$$0.00003123 = 0.00003\underset{\wedge}{123} = 3.123 \times 10^{-5}$$

Practice: Write the following in scientific notation. Watch the signs.
a) $0.000000043 =$ $0.00000254 =$ $0.000023 =$
b) $0.0000235 =$ $0.00007 =$ $0.0000000001 =$
c) $0.000000000028 =$ $0.00000000000009 =$ $0.00001 =$

Practice: Write the missing numbers.

Decimal Notation	**Scientific Notation**
d)	The population of the United States is 3×10^{8}.
e) The population of the world is about 6230,000,000.	
f)	The human heart is 3.1×10^{0} inches in diameter.
g) The diameter of an artery is .00621 inches.	
h)	The moon is 3.8×10^{5} km away.
i) The mass of the moon is 73,500,000,000,000,000,000,000.	
j)	The mass of the Earth is 5.97×10^{24} kg.
k)	A hair grows 2.08×10^{-4} mm per minute.

Multiplication and Scientific Notation

To multiply 3.1 x 10^4 X 2.3x10^5, use the commutative property 3.1x2.3x10^4x10^5 and the **associative property** (grouping of multiplication doesn't matter) to get 7.13x10^4x10^5. Then use the exponent rules to get the final result 7.13x10^9.

$(3x10^8)(1.5x10^{23})=4.5x10^{31}$ $(2.1x10^4)$ $(4x10^{-12})$ $=8.4x10^{-8}$ $(4.6x10^8)$ $(5.8x10^6)$
$=26.68x10^{14}$

Notice: What is wrong with the last example? Although the answer is correct, the number is not in scientific notation. To finish the problem, move the decimal *one space left* and *increase the exponent by one*.
$26.68x10^{14}$ = $2.668x10^{15}$

If the decimal moves right, the exponent moves down. →↓ $0.00042x10^9=4.2x10^5$
If the decimal moves left, the exponent moves up. ←↑ $7890x10^3=7.89x10^6$

Practice: Multiply then write the following in scientific notation.

a) $(3\times10^5)(2.3\times10^9)=$ $(2.1\times10^5)(1.3\times10^9)(2\times10^{10})=$ $(7.2\times10^5)(1\times10^9)=$

b) $(1.2\times10^{15})(5.3\times10^{15})=$ $(3.4\times10^{51})(2.3\times10^{-23})=$ $(3\times10^5)(2.3\times10^{-9})=$

c) $(-2\times10^{-9})(3\times10^9)=$ The negative in front of the 2 makes a number less than zero. The negative in front of the exponent makes a small number. $(-2.5\times10^{-5})(3\times10^{-12})=$

The following will need to have the decimal moved and the exponent adjusted after the multiplication.

d) $(-5.4\times10^{-7})(3.1\times10^8)=$ $(3.25\times10^{-9})(5.3\times10^{-12})=$ $(-2\times10^{12})(-7.3\times10^{23})=$

e) $(5.48\times10^{-15})(3.2\times10^{-10})=$ $(-7.2\times10^{21})(-4.3\times10^{35})=$ $(-2.5\times10^{-4})(-8.3\times10^{-8})=$

f)   The population of the world is 6.23x10^9 and the average person consumes 5x10^3 grams of sugar per year. How many grams of sugar are consumed in the world per year?

Division and Scientific Notation

Division works in a similar way. $\dfrac{4.5\times10^8}{3\times10^5}=\dfrac{4.5}{3}\times10^{8-5}=1.5\times10^3$

$\dfrac{1.092\times10^{-13}}{2.1\times10^{-5}}=\dfrac{1.092}{2.1}\times10^{-13-(-5)}=0.52\times10^{-8}=5.2\times10^{-9}$ Notice the decimal and exponent.

$\left(6.8\times10^{-3}\right)\div\left(3.4\times10^{-8}\right)=\dfrac{6.8\times10^{-3}}{3.4\times10^{-8}}=2\times10^{-3--8}=2\times10^5$

Practice: Divide the following then write the answer in scientific notation.

a) $\dfrac{1.5\times10^{21}}{2.3\times10^5}=$ $\dfrac{5\times10^{-71}}{2.3\times10^{52}}=$ $\dfrac{8.3\times10^{-7}}{1.2\times10^{-5}}=$

b) $\left(1\times10^{-3}\right)\div\left(1\times10^{-8}\right)=$ $\left(1\times10^3\right)\div\left(3.4\times10^8\right)=$ $\left(6.8\times10^{-3}\right)\div\left(3.4\times10^{-8}\right)=$

c) A certain string of bacteria is 3.4×10^{-5} cm long. There are approximately 80 cells in a string How wide is each single bacteria?

Calculator notation

Some calculators show scientific notation the same way we have written them here.
Some use EE. They write 3.456 EE5 to mean 3.456×10^5.
Some write the same thing with this in the screen 3.456^{05}.
There are several ways electronic equipment displays scientific notation. Look in your user's manual to see how it is displayed on your calculator.