2-8-1 Order of Operations

Look at both of the problems. Notice the difference in the way they are solved.

$48 \div 6 \times 2$
$= 8 \times 2$
$= 16$

The only difference in the way these problems were done is the order the operations were performed.

The one on the left is correct.

$48 \div 6 \times 2$
$= 48 \div 12$
$= 4$

Multiplication and division are always done left to right.
Remember multiplication is commutative and associative, but division is not. You can do problems that only contain multiplication in any order, but if division is in the problem, then the order **is** important.

$81 \div 3 \times 5$
$= 27 \times 5$
$= 135$

$4 \times 21 \div 2$
$= 84 \div 2$
$= 42$

$144 \div 6 \times 2 \div 3 \times 4$
$= 24 \times 2 \div 3 \times 4$
$= 48 \div 3 \times 4$
$= 16 \times 4 = 64$

Practice:

a) $45 \div 3 \times 2 =$ $\quad 2 \div 6 \div 3 \times 9 =$ $\quad \left(\frac{2}{3}\right)\left(\frac{5}{6}\right) \div \left(\frac{3}{4}\right) =$ $\quad 4\frac{2}{5} \div 1\frac{1}{2} \times 5\frac{1}{3} =$

b) $5 \div 10 \times 200 =$ $\quad 45 \times 2 \div 5 =$ $\quad 45\frac{}{3} \div \frac{3}{9} \times \frac{2}{5} =$ $\quad -8 \div (-4) \times (-7) =$

c) $4\frac{1}{2} \div 3 \times (-2) =$ $\quad 510 \div 5 \div 12 =$ $\quad 42 \times (-3) \div (9) =$ $\quad 4 \times (-1) \div (-5) \times (-24) =$

Addition and subtraction work the same way. Subtraction isn't commutative. Remember to think "add the opposite" when subtracting, but do it left to right.

Practice:

d) $45 - 3 + 2 =$ $\quad 2 - 6 - 3 + 9 =$ $\quad \left(\frac{2}{3}\right) + \left(\frac{5}{6}\right) - \left(\frac{3}{4}\right) =$ $\quad 4\frac{2}{5} - 1\frac{1}{2} + 5\frac{1}{3} =$

e) $\left(\frac{1}{3}\right) + \left(\frac{1}{8}\right) - \left(\frac{1}{4}\right) =$ $\quad \left(\frac{5}{8}\right) - \left(\frac{1}{5}\right) + \left(\frac{3}{10}\right) =$ $\quad 4\frac{1}{2} - 3\frac{2}{3} + 6 =$ $\quad 8 - \frac{2}{3} + 7\frac{3}{4}$

Multiplication and division are always done before addition and subtraction. Write each step out completely under the previous step.

$3 \cdot 5 - 7 \cdot 8 =$ $15 - 56 =$ -41	Multiplication before subtraction. Note: Write the new problem after multiplying.	$-8 \cdot 5 + 7(-8) =$ $-40 - 56 =$ -96	$8 \div 4(-6) - 3 + 5$ $2(-6) - 3 + 5$ $-12 - 3 + 5$ $-15 + 5 = -10$

Practice:

f) $45 + 3 \times 2 =$ $\quad 2 - 6 \div 3 \times 9 =$ $\quad \left(\frac{2}{3}\right) - \left(\frac{5}{6}\right) \div \left(\frac{3}{4}\right) =$ $\quad 4\frac{2}{5} \div 1\frac{1}{2} - 5\frac{1}{3} =$

g) $4 \cdot 5 + 3 \cdot 2 =$ $\quad 4 \div 5 - 3 \times 2 =$ $\quad 12 \div (-4) + 5(-2) =$ $\quad \left(\frac{2}{3}\right)(-9) - 3 \times (-2) =$

There is a mistake in each of the following problems. Discover what was done **incorrectly**.

h) $12-4\cdot 8=$
$8\cdot 8=64$
-20 is correct.

$9\div 12\times 3=$
$9\div 36=$
$\frac{1}{4}$
2 ¼ is correct.

$15-3+5=$
$15-8=$
7
17 is correct.

Exponents are done before multiplication and division.

$3\cdot 2^3-7^2=$
$3\cdot 8-49=$
$24-49=$
-25

$4^3\div 8-3^2\cdot 2+5=$
$64\div 8-9\cdot 2+5=$
$8-18+5$
$-10+5=-5$

Note in the examples, the exponents are done and the rest of the problem is written down.
If you scratch to the side and skip writing all steps, you will make mistakes.

A root is an exponent and also done before multiplication, division, addition or subtraction.

$(½)^5=(½)(½)(½)(½)(½)={}^1/_{32}$ $(-{}^3/_4)^3=(-{}^3/_4)\ (-{}^3/_4)\ (-{}^3/_4)=-{}^{27}/_{64}$ $3-4\sqrt{64}=3-4(8)=3-32=-29$

Practice:

i) $5^2+3\times 2^2=$ $2^3-6\div 3\times 3^2=$ $\left(-\frac{2}{3}\right)^2-\left(\frac{5}{6}\right)\div\left(\frac{3}{4}\right)^2=$ $4\frac{2}{5}\div\left(1\frac{1}{2}\right)^3-5\frac{1}{3}=$

j) $3^2(2)-5\times 2^2=$ $4^2+5\times(-2)^3=$ $50\div 2+5\times 2^2=$ $15^2-3\times 5^2=$

The first thing always done is parenthesis or other grouping symbols.
If there are nested grouping symbols, work from the inside out.

$4+2(7-3(8+6))=$ Work the inside parenthesis first.
$4+2(7-3(14))=$
$4+2(7-42)=$ The next set of parenthesis has two operations inside.
Always do multiplication before subtraction.
$4+2(-35)=$ Finish the inside of the second parenthesis.
$4-70=-66$ Multiply before subtracting.

Absolute value, division bar and root symbols also act like parenthesis.

$\frac{5-8\cdot 7}{2^3-5}=$
$\frac{5-56}{8-5}=$
$\frac{-51}{3}=-17$

The division bar groups the operations.

In the numerator the multiplication is done before subtraction.

In the denominator the exponent is done first.

Finally the division is done.

$8-\sqrt{3^2+5(3\cdot 5-2^2)}=$
$8-\sqrt{3^2+5(3\cdot 5-4)}=$
$8-\sqrt{3^2+5(15-4)}=$
$8-\sqrt{3^2+5(11)}=$
$8-\sqrt{9+5(11)}=$
$8-\sqrt{9+55}=$
$8-\sqrt{64}=8-8=0$

The first step in the inner parenthesis is the exponent.
Then the multiplication is done.

The subtraction finishes the inner parenthesis.
The root acts like a parenthesis.
The exponent is done before the multiplication.
The addition is done before the root is taken.

Finally the subtraction is done.

To remember the order of operations use the mnemonic devise **Please excuse my dear Aunt Sally**.

Please	**P**arenthesis and other grouping symbols. ( ), { }, [ ], $\sqrt{\ }$, $-$, $\lvert\ \rvert$	$3(4-5)=$ The subtraction is done first 3(-1)=-3 because it is in the parenthesis.
Excuse	**E**xponents This includes roots. x^2 or $\sqrt{s}$	$14-4^3=$ The exponent is done before the subtraction. 14-64=-50
My **D**ear	**M**ultiply and **D**ivide from left to right. 3x5, $3\cdot 5$ and 3(5) all mean multiply. $12\div 4$, $\frac{12}{4}$, and $4\overline{)12}$ are all the same division.	$4\cdot 6\div 8=24\div 8=3$ $72\div 3\times 4=24\times 4=96$
Aunt **S**ally	**A**dd and **S**ubtract from left to right.	4-5+8=-1+8=7

One more extra fun example:

$\frac{-7\pm\sqrt{7^2-4(2)(-30)}}{2(2)}=\frac{-7\pm\sqrt{49-4(2)(-30)}}{2(2)}=\frac{-7\pm\sqrt{49-8(-30)}}{2(2)}=$ $\frac{-7\pm\sqrt{49+240}}{2(2)}=\frac{-7\pm\sqrt{289}}{2(2)}=\frac{-7\pm 17}{2(2)}=\frac{-7\pm 17}{4}$	7^2=49, then 4(2)=8, then 8(30)=240, next 49+240=289. The root is 17.

From this point there are two answers. Split the problem into two.

For the minus 17 use $\frac{-7-17}{4}=-6$. For the plus 17 use $\frac{-7+17}{4}=\frac{10}{4}=5/2$.

Practice:

a) $25-8(3+2)$ $\quad 5+8(3+2)$ $\quad 3\div 6\times 18$

b) $5-8(-3-2)$ $\quad -3(-3)+9(7-8)$ $\quad 4/5+(3/2)(2/5)$

c) $3\frac{2}{3}(2\frac{5}{11})-4\div 5$ $\quad 81\div 6+3(4-7)$ (Fraction answer) $\quad \frac{3}{4}-\frac{1}{2}\left(3\div 4+\frac{2}{3}\right)$

d) $12-107+89-(-72)$ $\quad 3\frac{3}{4}-8\frac{2}{5}+1\frac{3}{5}$ $\quad \left(-3\frac{1}{3}\right)\left(4\frac{4}{9}\right)\div(-2)$

e) $3.4-1.7+0.9-(-7.2)$ $\quad 1\frac{2}{5}-5\frac{2}{5}+1\frac{3}{5}$ $\quad \left(-5\frac{1}{3}\right)\left(3\frac{3}{4}\right)\div(-2)$

f) $-7-(-8)+9(3-14)$ $\quad -9\frac{3}{4}+(-3\frac{2}{4})-(-1\frac{3}{5})$ $\quad \left(-3\frac{1}{3}\right)\div\left(4\frac{4}{9}\right)\times(-2)$

g) $\frac{(7-4)^3+3}{50-6\cdot 3}$ $\quad \frac{\left(7\frac{1}{3}-2\frac{5}{6}\right)^2+\frac{1}{2}}{2\frac{2}{5}-\left(\frac{5}{6}\right)\cdot\left(\frac{2}{5}\right)}$ $\quad \frac{(7)^3-\left(\frac{1}{2}\right)^3}{3-(5+8)}$

h) $\frac{(9-12)^4+5}{5-6(-3)}$ $\quad \frac{\left(5\frac{1}{3}-2\frac{3}{4}\right)^2+\frac{3}{4}}{1\frac{2}{3}\left(\frac{1}{4}\right)\left(\frac{2}{5}\right)}$ $\quad \frac{(3)^4-\left(\frac{3}{4}\right)^2}{3-(5-8)}$

i) $\frac{4+3^2-6}{7}+8(2+5)$ $\quad \frac{\frac{2}{3}+\left(\frac{3}{2}\right)^2-6}{\frac{2}{3}}+\frac{1}{2}(2+8)$ $\quad \frac{\frac{1}{4}\div\left(\frac{3}{2}\right)^2-\frac{1}{4}}{\frac{2}{5}+\frac{1}{3}}+\frac{2}{3}(2+\frac{4}{5})$

j) $\sqrt{2+3\cdot 5-(-8)^3}$ $\quad 2-8|7-5\cdot 3|$ $\quad 3-8\sqrt{3\cdot 10^2-3\cdot 5^2}$

k) $-8-\dfrac{3-(8-56)}{6-9}$ $\quad -12-\dfrac{3\frac{2}{3}+5\sqrt{\frac{4}{9}}}{\frac{1}{3}}$ $\quad \sqrt{\dfrac{9-3(2)}{3\cdot 2^2}}-7$

l) $-5^2=-25$ $\quad -12^2=$ $\quad -9^2=$

Exponent before negative

m) $(-5)^2=25$ $\quad (-12)^2=$ $\quad (-9)^2=$

Parenthesis before exponent

n) $-2^2=$ $\quad -\left(\frac{2}{3}\right)^2=$ $\quad (-3)^2=$

o) $(-15)^2=$ $\quad -15^2=$ $\quad (-2.1)^2=$

p) $-2.1^2=$ $\quad \left(-\frac{2}{3}\right)^2=$ $\quad -3^2=$

q) $\dfrac{3^2-5^2}{7-(8-3\cdot 5)}$ $\quad -8^2+(-5)^2$ $\quad \dfrac{-8\pm\sqrt{8^2-4\cdot 5\cdot 3}}{2(5)}$

r) $\frac{1}{2}(5)(8+2)$ $\quad 2(3.14)(5)^2+2(3.14)(5)(7)$ $\quad 4\left(\frac{1}{2}(3)(8)\right)+3^2$

s) $\dfrac{5(82-32)}{9}$ $\quad \dfrac{9(28)}{5}+32$ $\quad \dfrac{5(3-32)}{9}$

t) $\dfrac{9(-8)}{5}+32$ $\quad 1200\left(1+\dfrac{.15}{4}\right)^{(4)(5)}$ $\quad 8{,}000\left(1+\dfrac{.05}{12}\right)^{(12)(3)}$

u) $\dfrac{-14\pm\sqrt{14^2-4(6)(-40)}}{2(6)}$ $\quad \dfrac{-(-3)\pm\sqrt{(-3)^2-4(2)(-20)}}{2(2)}$ $\quad \dfrac{-1\pm\sqrt{1^2-4(1)(-6)}}{2(1)}$