3-4-2 Ratio

A **ratio** is a comparison of two numbers.

How much sugar do you put in your favorite cookie recipe? How much flour?

In mine there are 2 cups of sugar and 5 cups flour, so the ratio of sugar to flour is 2 to 5.

I can write this 2 to 5 or 2:5 or $\frac{2}{5}$.

The ratio of flour to sugar is 5 to 2 or 5:2 or $\frac{5}{2}$.

The last example looks like a fraction, but **don't** change a ratio to a mixed number (2 ½). Remember a ratio is the comparison of two numbers so keep them in fraction form.

Practice:

a) What is the ratio of one foot to inches? What is the ratio of inches to one foot?

b) What is the ratio of men to women in your class or at work?

c) Simplify the ratio of 12:15

d) A class of 20 students has 12 women. (Reduce)

What is the ratio of the number of men to the total number of students?

What is the ratio of the number of men to the number of women?

What is the ratio of the number of women to the number of men?

e) In a bucket there are 105 what-cha-ma-call-its and 45 doohickeys.

What is the ratio of the number of what-cha-ma-call-its to the total number of things in the bucket?

What is the ratio of the number of doohickeys to the total number of things in the bucket?

What is the ratio of the number of doohickeys to the number of what-cha-ma-call-its?

f) From a total yearly budget of $24,000,000, Special City spends $4,000,000 on education. What is the ratio of the amount spent on education to the amount not spent on education?

g) A test of 50 questions included 15 ratio problems and 5 percentage problems. What is the ratio of the total number of fraction and decimal problems to the number of problems on a test covering only fractions, decimals, percentages, and proportions?

Proportion

A **proportion** is two equal ratios. If I want to use 15 cups of flour, how much sugar do I need?

$\frac{\text{Flour}}{\text{Sugar}}\ \frac{5}{2}=\frac{15}{x}$ When this is written in fraction form, you may just reduce or raise the fraction to find the answer. I would need 6 cups of sugar for the cookie recipe.

Another way to write the same question is 5:2::15:x

This is read "five is to 2 as 15 is to x."
":" is read "is to" and "::" is read "as."

The ratio of men to women in my building is 4 to 3. If there are 15 women, how many men?
Write 4:3::x:15 or write $\frac{\text{men}}{\text{women}}\ \frac{4}{3}=\frac{x}{15}$ Notice that the x is across from the label "men."

Sometimes we can't easily raise the fraction to get the answer. There is a process that works all the time.
We **cross multiply**. $\frac{\text{men}}{\text{women}}\ \frac{4}{3}=\frac{x}{15}$ becomes $4\cdot 15=3x \Rightarrow 60=3x \Rightarrow 60\div 3=20$ so $x=20$

One way to think of this is to multiply the given diagonal numbers. Then divide by the remaining number. In the example, multiply 4 times 15 then divide by 3 to get 20.

Practice:

$\frac{21}{14}=\frac{51}{x}$ becomes $14\cdot 51=21x \Rightarrow 714=21x \Rightarrow 714\div 21=34=x$

a) $\frac{2}{3}=\frac{28}{x}$ or 2 :3 :: 28 :x $\frac{x}{10}=\frac{8}{5}$ or ___ : ___ :: ___ :___ ___ = ___ or 35 :8 :: x :16

b) $\frac{5}{2}=\frac{8}{x}$ or ___ : ___ :: ___ : ___ $\frac{123}{57}=\frac{x}{3}$ or ___ : ___ :: ___ :___ ___ = ___ or 3:7 :: x : 1

$\frac{5.1}{1.4}=\frac{2}{x}$ becomes $1.4\times 2=5.1x \Rightarrow 2.8=5.1x \Rightarrow 2.8\div 5.1=0.549=x$ (Rounded to the nearest thousandth)

c) $\frac{1.2}{2}=\frac{8.7}{x}$ or ___ : ___ :: ___ : ___ $\frac{1}{x}=\frac{2}{3.1}$ or ___ : ___ :: ___ : ___ ___ = ___ or 3.5:8 :: x : 1

$\frac{2\frac{1}{2}}{4}=\frac{1}{x}$ becomes $4\times\frac{1}{3}=2\frac{1}{2}x \Rightarrow \frac{4}{3}=2\frac{1}{2}x \Rightarrow \frac{4}{3}\div 2\frac{1}{2}=\frac{4}{3}\div\frac{5}{2}=\frac{4}{3}\times\frac{2}{5}=\frac{8}{15}=x$

d) $\frac{3}{x}=\frac{5/7}{2/5}$ $\frac{x}{2\frac{1}{3}}=\frac{3\frac{3}{4}}{1/8}$ $\frac{1/2}{5}=\frac{x}{2/5}$

e) $\frac{3}{x} = \frac{0.4}{1\frac{1}{2}}$ $\frac{x}{2\frac{1}{3}} = \frac{3}{1\frac{}{}/5}$ $\frac{0.5}{5} = \frac{x}{2\frac{}{}/5}$

f) $\frac{3}{x} = \frac{75}{100}$ $\frac{x}{2\frac{1}{4}} = \frac{25}{100}$ $\frac{1\frac{}{}/2}{5} = \frac{x}{100}$

g) $\frac{0.2}{x} = \frac{3}{100}$ $\frac{x}{2} = \frac{3\frac{3}{4}}{100}$ $\frac{3\frac{}{}/8}{5} = \frac{x}{100}$

h) $\frac{4}{x} = \frac{5\frac{}{}/3}{5\frac{1}{}/3}$ $\frac{5}{2\frac{1}{3}} = \frac{3\frac{3}{4}}{x}$ $\frac{5.6}{4} = \frac{78}{x}$

Easy Proportion Problems

Use proportions to solve a wide variety of problems.

Watch for these words: per, rate, ratio, to. They may be a clue that the problem is easy to solve using a proportion.

Example: Grandma can ride the 24 miles to Blackfoot in two hours on her bike. If she rides at the same rate, how long will it take her to ride her bike the 36 miles to Rexburg?

Step one: What is asked? Underline the question. Grandma can ride the 24 miles to Blackfoot in two hours on her bike. If she rides at the same rate, how long will it take her to ride her bike the 36 miles to Rexburg?

Step two: What information is given? Highlight the information. Grandma can ride the 24 miles to Blackfoot in two hours on her bike. If she rides at the same rate, how long will it take her to ride her bike the 36 miles to Rexburg?

Step three: Is a ratio given? if so set up the problem with three lines and an equals. ——— – = –

Step four: Fill in the labels. At least one label comes from the question. Grandma can ride the 24 miles to Blackfoot in two hours on her bike. If she rides at the same rate, how long will it take her to ride her bike the 36 miles to Rexburg? "How long" says to use "hours" for one label. It works most of the time to use a label from the number closest to the question. "36 miles" is in the question sentence so use "miles" for the other label. $\frac{\text{hours}}{\text{miles}} = -$

Step five: Fill in numbers. Put the given ratio on one side of the equal sign. Put the other piece of information on the same level as its label. Put an "x" in the empty space. $\frac{\text{hours}}{\text{miles}} \quad \frac{2}{24} = \frac{x}{36}$

Step six: Solve the proportion. $\frac{\text{hours}}{\text{miles}} \quad \frac{2}{24} = \frac{x}{36} \Rightarrow 2 \cdot 36 = 24x \Rightarrow x = 3 hours$

Step seven: Ask yourself if your answer is plausible.

Try one:

Kathy uses 3 cups of sugar to make 48 chocolate chip cookies. She has five cups of sugar in her sugar canister. How many cookies can she make?

Step one: Underline the question.

Step two: Highlight the information.

Step three: How do you know to make a proportion?

Step four, five and six:

Does your answer seem correct? If she can make 48 cookies with 3 cups of sugar would she make more or less with 5 cups of sugar?

Practice:

a) I run four miles around the lake in 45 minutes. If I only have 30 minutes to run before work, how far can I expect to run? $\frac{\text{miles}}{\text{minutes}} \quad \frac{\quad}{\quad} = \frac{\quad}{\quad}$

b) Jonathan drives an average of 52 miles per hour. How far can he drive in 3 ½ hours? Hint: 52 miles in how many hours?

c) Tom loses four pounds a week on a diet. He diets for 20 weeks. How much weight did he lose?

d) Sandy can get 35 miles to the gallon in her new car. How many gallons of gas will it take her to drive 525 miles to visit her sister?

e) The Munchkin Auto Team averages one win in every five bowling games. If they play 15 games this winter, how many do you expect they will win? How many do you expect them to lose?

f) Steve can drive 182 miles to deliver a car in three hours. If he drives at the same speed, how long will it take him to drive the 1098 miles on the next trip?

g) Six pounds of grass seed covers 150 square feet. My lawn is 200 square feet. How much seed do I need?

h) Christmas fudge uses one and a half pounds of powdered sugar to make 64 pieces. How many pounds of sugar do we need to make 192 pieces of microwave fudge?

i) Toothless Car Wash (the kid next door) can wash 3 cars in one and half hours. How long will it take to wash 10 cars?

j) Brian scored 28 points the first half of a game. If he continues at this rate how many points will he have in a season of 10 games?

k) There are 2.54 centimeters in one inch. How many centimeters in 15 inches?

l) Use the conversion factor 2.54 centimeters = 1 inch. How many inches in 15 centimeters?

m) 1 foot = 12 inches How many feet in 42 inches?

n) We pack 142 crates per day. At this rate how long will it take to finish a 1000 crate job?

o) There are 144 in^2 in 1 ft^2. How many square inches are in 15 square feet?

p) If a car gets 21 miles to the gallon, how far will it go on a full 10 gallon tank?

Unit Conversion Proportion Problems

Use proportions to solve some unit conversion problems. Use a conversion chart to get the ratio. Then set up labels, fill in numbers and solve for the desired unit.

Example: We ran 100 meters. How far is that in feet?

Step one: What is asked? Underline the question. We ran 100 meters. How far is that in feet?

Step two: What information is given? Highlight the information. We ran 100 meters. How far is that in feet?

Step three: Is a ratio given? If so, set up the problem with three lines and an equal sign.

———— – = – Look up on a chart the ratio of meters to feet.

Conversion Factors

Length:
1 inch = 25.4 millimeters
1 inch = 2.54 centimeters
1 foot = 0.3048 meters
1 mile = 1.609 kilometers
1 millimeter = 3.937E-2 inches
1 centimeter = 0.3937 inches
1 meter = 3.281 feet

1 mile = 1760 yards = 5280 feet

1 yard = 3 feet = 36 inches

1 foot = 12 inches

Use either conversion. The answer will be the same within a few decimal places.

Step four: Fill in the labels. At least one label comes from the question. We ran 100 meters. How far is that in feet?

"How far" says to use "feet" for one label. Use meters for the other label.

$$\frac{\text{meters}}{\text{feet}} \quad \frac{100}{x} = \frac{1}{3.281}$$

$$\frac{\text{meters}}{\text{feet}} \quad \frac{100}{x} = \frac{0.3048}{1}$$

Step five: Fill in numbers. Put the given ratio on one side of the equal sign. Put the other piece of information on the same level as its label. Put an "x" in the empty space.

Step six: Solve the proportion. 328.1 feet or 328.0839895 feet Round this to 328.1 feet. Notice that either conversion factor gives the same, or close enough, answer.

Step seven: Ask yourself if your answer is plausible.

Try one: How many inches in 120 centimeters.

Step one: Underline the question.
Step two: Highlight the information.
Step three: How do you know to make a proportion?

Step four, five and six: Does your answer seem correct?

Practice Questions:

Use the conversion chart at the end of this assignment.

a) How many inches in 23 cm?

b) How many inches in 0.7 cm?

c) How many centimeters in 5 inches?

d) How many centimeters in 0.45 inches?

e) How many centimeters in 23 inches?

f) How many feet in 43 meters?

g) How many meters in 2.5 feet?

h) How many feet in 0.5 meters?

i) How many miles in 42 km?

j) How many kilometers in 42 miles?

k) How many kilometers in 10 miles?

l) How many yards in 42 feet?

m) How many miles in 300 feet?

n) How many feet in 12 miles?

o) How many square inches in 34 square centimeters?

p) How many square feet in 34 square meters?

q) How many square feet in 34 square yards? (remember 9 sqft= 1sqyrd)

r) How many acres in 345 hectares?

s) How many square meters in 1.5 square feet?

t) How many square miles in 14 hectares?

u) How many liters in 15 quarts?

v) How many liters in 4.2 quarts?

w) How many quarts in 10 liters?

x) How many quarts in 0.45 liters?

y) How many cubic feet in 10 liters?

z) How many gallons in 5 liters?

aa) How many liters in 5 cubic yards?

ab) How many gallons in 15 cubic feet?

ac) Challenge: How many meters in 15 inches?

ad) Challenge: How many yards in 15 meters?

Conversion Factors

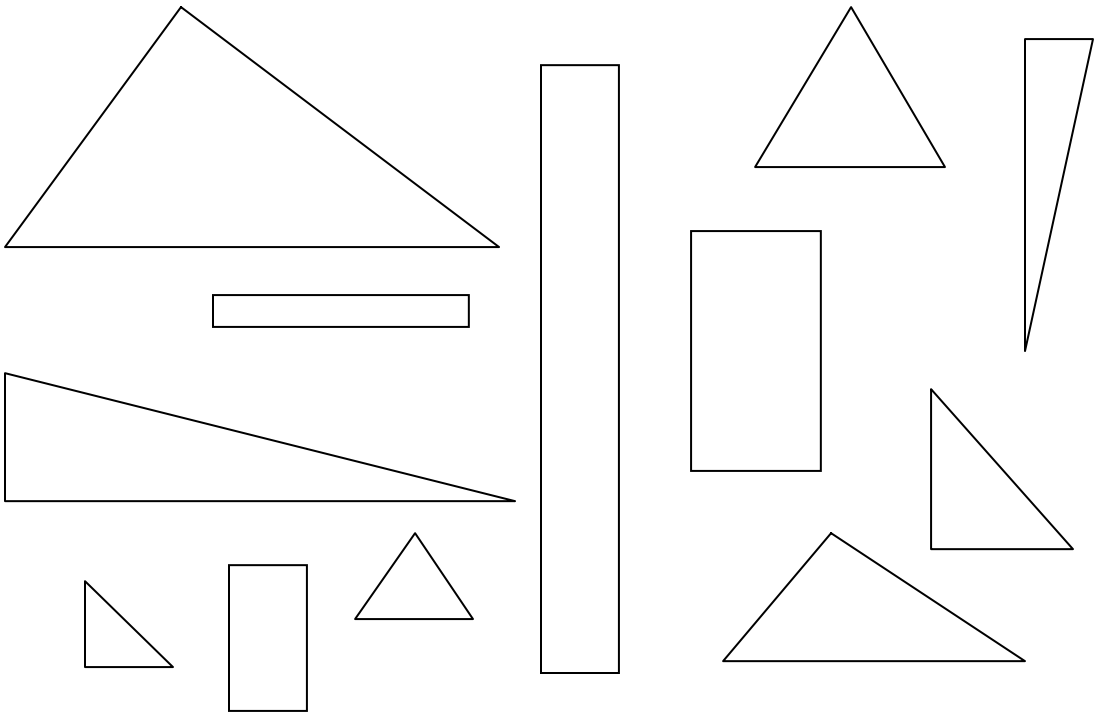
Length:
1 inch = 25.4 millimeters
1 inch = 2.54 centimeters
1 foot = 0.3048 meters
1 mile = 1.609 kilometers
1 millimeter = 0.03937 inches
1 centimeter = 0.3937 inches
1 meter = 3.281 feet
1 mile = 1760 yards = 5280 feet
1 yard = 3 feet = 36 inches
1 foot = 12 inches

Area:
1 sq inch = 6.452 sq centimeters
1 sq foot = 0.0929 sq meters
1 sq mile = 2.59 sq kilometers
1 sq mile = 259 hectares
1 acre = 0.4047 hectares
1 sq centimeter = 0.1550 sq inches
1 sq meter = 10.77 sq feet
1 sq kilometer = 0.3861 sq miles
1 hectare = 0.003861 sq miles

Volume:
1 liter = 1.06 quart
1 cu inches = 16.39 cu centimeters
1 cu foot = 0.02832 cu meters
1 cu foot= 7.481 gallons (US)
1 cu foot= 28.32 liters
1 cu yard = 0.7646 cu meters
1 gallon (US) = 3.785 litres
1 gallons (US) = 0.003785 cu metres
1 cu centimetres = 0.06102 cu inches
1 cu meters = 35.31 cu feet
1 gallons (US) = 0.1337 cu feet
1 litres = 0.03531 cu feet
1 cu meter = 1.308 cu yards

Proportion Problems Involving Similar Shapes

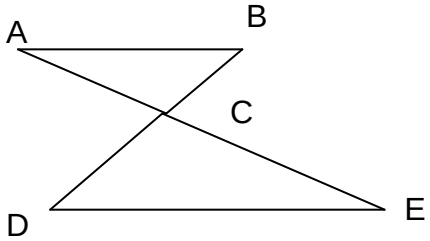
Two shapes are similar if they have the same shape, but not the same size. Below are six pairs of similar shapes. Match them up.



The ratio of dimensions in one shape will be the same as the ratio in a similar shape. This can used to make a proportion.

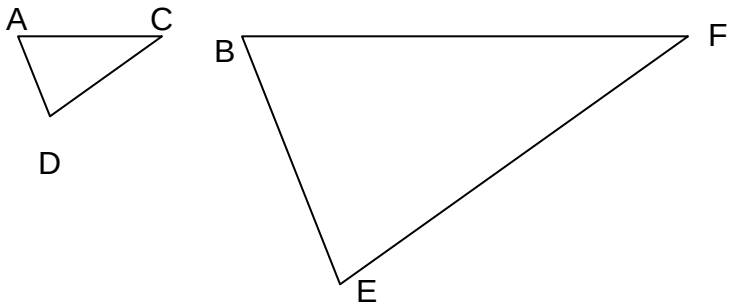
Practice:

a) If line AB is 12 inches and line BC is 7 inches, what is the measure of line DC if line DE is 48 inches?



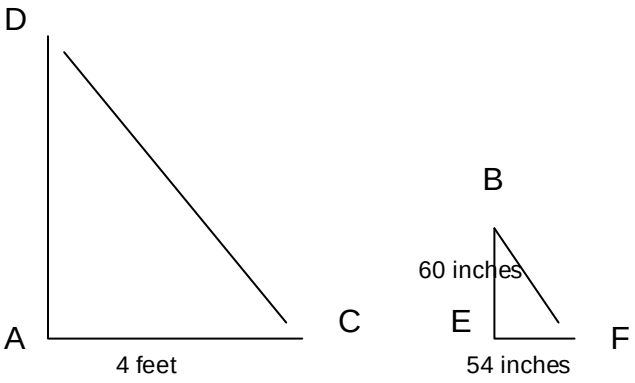
If we use the labels longest and shortest in a proportion, we will have two equal ratios and will be able to solve for the length of DC.

$$\frac{longest}{shortest} \quad \frac{12}{7} = \frac{48}{x}$$



b) AD is 20 feet long. AC is 30 feet long.
CD is 25 feet long.

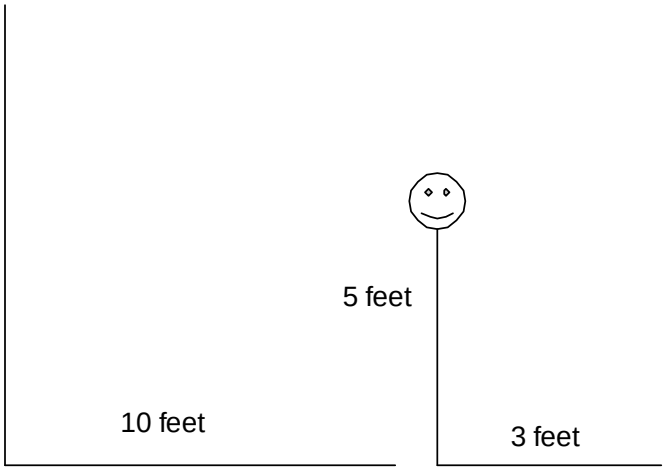
How long is EF if BE is 105 feet long?



c) The line from D to C might not be drawn in the problem. You need to visualize it or draw it in.

Find the length of DA.

Example: A flagpole casts a 10 foot shadow. Five foot Charlie casts a 3 foot shadow at the same time of day. How tall is the flag pole?



Step one: What is asked? Underline the question. A flag pole casts a 10 foot shadow. Five foot Charlie casts a 3 foot shadow at the same time of day. How tall is the flag pole?

Step two: What information is given? Highlight the information. Draw a picture. A flag pole casts a 10 foot shadow. Five foot Charlie casts a 3 foot shadow at the same time of day. How tall is the flag pole?

———— — = — Step three: Is a ratio given? if so set up the problem with three lines and an equals.

Step four: Fill in the labels. At least one label comes from the question. A flag pole casts a 10 foot shadow. Five foot Charlie casts a 3 foot shadow at the same time of day. How tall is the flag pole?

"Height" would be one label. It works most of the time to use a label from the number closest to the question. "3 foot shadow" is in the question sentence so use "shadow" for the other label.

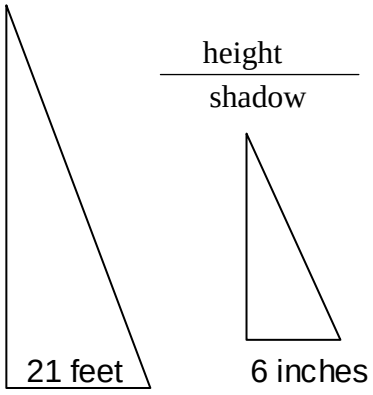
$$\frac{\text{height}}{\text{shadow}} = —$$

Step five: Fill in numbers. Put the given ratio on one side of the equal sign. Put the other piece of information on the same level as its label. Put an "x" in the empty space.

$$\frac{\text{height}}{\text{shadow}} \quad \frac{5}{3} = \frac{x}{10}$$

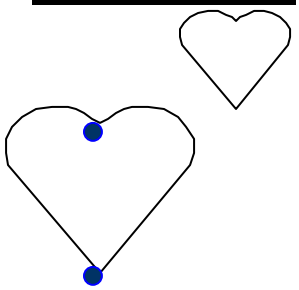
Step six: Solve the proportion.

Step seven: Ask yourself if your answer is plausible.



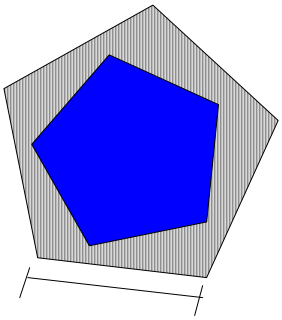
d) The plans for a tower include a scale diagram. The tower has a wire from the top to the ground 21 feet from the bottom of the tower. On the drawing this wire is 6 inches from the bottom of the tower. The tower is 8 inches tall and the wire is 10 inches long. How tall is the actual tower, and how long is the actual wire?

e) The 3 by 5 picture shown at the left is to be enlarged. The short side will measure 20 inches. What will be the measurement of the long side?



f) the length from point to point (dot to dot) on the large heart is 15 inches and the perimeter (length around the outside) of the large heart is 108.5 inches. If the length from point to point on the small heart is 7 inches, what is the perimeter?

g) The ratio of the large pentagon to the small one is 1 to ¾. What is the length of the side of the small pentagon?



12 miles

h) In the following figure there are 3 sets of similar shapes. Find the indicated length.