

6-1 Multiplication of Binomials

A binomial is two terms. $3x-5$ is a binomial with terms $3x$ and -5 .

To learn how to multiply binomials, first review the steps for multiplying two digit numbers.

$$\begin{array}{r} 23 \\ 12 \\ \hline 46 \\ 23 \\ \hline 276 \end{array}$$

Multiply the first digit of the bottom number by each of the digits of the top number.

Shift over to line up the place value. Multiply the second digit of the bottom number by both digits in the top number.
Add.

$$\begin{array}{r} 2x - 3 \\ x + 2 \\ \hline 4x - 6 \\ 2x^2 - 3x \\ \hline 2x^2 + x - 6 \end{array}$$

Multiply the first term of the bottom binomial number by each of the terms of the top binomial.

Shift over to line up the like terms. Multiply the second term of the bottom binomial by each of the terms in the top binomial.
Combine like terms.

Notice the steps for multiplication of binomials are the same as for multiplying two digit numbers.

This works because of the distributive property. $(2x-3)(x+2)=(2x-3)x+(2x-3)2=2x^2-3x+4x-6$
You don't have to work the problems using distributive property. Normally binomials are multiplied horizontally instead of vertically using what we call the FOIL method.

| $(2x - 3)(x + 2)$ | | | | | $(3x-5)(5x-8)$ | | | |
|-------------------------------------|----------------|--------------|------------|--|----------------------------------|---------------|--------------|------------|
| First terms | Outside terms | Inside terms | Last terms | | First terms | Outside terms | Inside terms | Last terms |
| $(2x)(x)$ | $(2x)(2)$ | $(-3)(x)$ | $(-3)(2)$ | | $(3x)(5x)$ | $(3x)(-8)$ | $(-5)(5x)$ | $(-5)(-8)$ |
| F | O | I | L | | F | O | I | L |
| $2x^2$ | $+4x$ | $-3x$ | -6 | | $15x^2$ | $-24x$ | $-25x$ | $+40$ |
| | $2x^2 + x - 6$ | | | | $15x^2 - 49x + 40$ | | | |

Practice:

| $(x - 5)(x - 5)$ | | | | | $(3x-5)(5x+8)$ | | | |
|------------------------------------|--------------------------------------|--------------|------------|--|----------------------------------|---------------|--------------|------------|
| First terms | Outside terms | Inside terms | Last terms | | First terms | Outside terms | Inside terms | Last terms |
| F | O | I | L | | F | O | I | L |
| | $(3x - 5)(3x + 5)$ | | | | $(x-2)(x-3)$ | | | |
| First terms | Outside terms | Inside terms | Last terms | | First terms | Outside terms | Inside terms | Last terms |
| F | O | I | L | | F | O | I | L |

Practice:

| | | | |
|-----------------------|--------------------|------------------|--------------------|
| a) $(x + 3)(x + 5)$ | $(v + 5)(v + 1)$ | $(x + 9)(x + 5)$ | $(y + 3)(y + 2)$ |
| b) $(t + 1)(t + 3)$ | $(x + 10)(x + 3)$ | $(x + 4)(x + 5)$ | $(y + 7)(y + 6)$ |
| c) $(x + 7)(x + 3)$ | $(x + 8)(x + 7)$ | $(x + 7)(x + 9)$ | $(x + 6)(x + 8)$ |
| d) $(x + 5)(x + 7)$ | $(x + 8)(x + 3)$ | $(x + 9)(x + 8)$ | $(x + 12)(x + 11)$ |
| e) $(x - 3)(x - 5)$ | $(v - 5)(v - 1)$ | $(x - 9)(x - 5)$ | $(y - 3)(y - 2)$ |
| f) $(t - 1)(t - 3)$ | $(x - 10)(x - 3)$ | $(x - 4)(x - 5)$ | $(y - 7)(y - 6)$ |
| g) $(x - 7)(x - 3)$ | $(x - 8)(x - 7)$ | $(x - 7)(x - 9)$ | $(x - 6)(x - 8)$ |
| h) $(x - 5)(x - 7)$ | $(x - 8)(x - 3)$ | $(x - 9)(x - 8)$ | $(x - 12)(x - 11)$ |
| i) $(x + 3)(x - 5)$ | $(v - 5)(v + 1)$ | $(x + 9)(x - 5)$ | $(y - 3)(y + 2)$ |
| j) $(t - 1)(t + 3)$ | $(x + 10)(x - 3)$ | $(x - 4)(x + 5)$ | $(y + 7)(y - 6)$ |
| k) $(x - 7)(x + 3)$ | $(x + 8)(x - 7)$ | $(x - 7)(x + 9)$ | $(x - 6)(x + 8)$ |
| l) $(x + 5)(x - 7)$ | $(x - 8)(x + 3)$ | $(x + 9)(x - 8)$ | $(x - 12)(x + 11)$ |
| m) $(r + 1)(r + 1)$ | $(x + 10)(x + 10)$ | $(x + 7)(x + 7)$ | $(y + 8)(y + 8)$ |
| n) $(x + 5)(x + 5)$ | $(z + 2)(z + 2)$ | $(m + 6)(m + 6)$ | $(x + 4)(x + 4)$ |
| o) $(y + 11)(y + 11)$ | $(x + 12)(x + 12)$ | $(x + 3)(x + 3)$ | $(x + 9)(x + 9)$ |
| p) $(r - 1)(r - 1)$ | $(x - 10)(x - 10)$ | $(x - 7)(x - 7)$ | $(y - 8)(y - 8)$ |
| q) $(x - 5)(x - 5)$ | $(z - 2)(z - 2)$ | $(m - 6)(m - 6)$ | $(x - 4)(x - 4)$ |
| r) $(y - 11)(y - 11)$ | $(x - 12)(x - 12)$ | $(x - 3)(x - 3)$ | $(x - 9)(x - 9)$ |
| s) $(r - 1)(r + 1)$ | $(x - 10)(x + 10)$ | $(x + 7)(x - 7)$ | $(y - 8)(y + 8)$ |
| t) $(x + 5)(x - 5)$ | $(z + 2)(z - 2)$ | $(m + 6)(m - 6)$ | $(x + 4)(x - 4)$ |

The next set of practice has leading coefficients other than one in one or both of the binomials. Some are challenging with exponents other than one.

$$(3x^2 + 5)(4x^3 - 7) = (3x^2)(4x^3) + (3x^2)(-7) + (5)(4x^3) + (5)(-7) = 12x^5 - 21x^2 + 20x^3 - 35$$

| | | | |
|----------------------|--------------------|-------------------|-------------------|
| a) $(4x + 3)(x + 5)$ | $(-5v + 5)(v + 1)$ | $(8x + 9)(x + 5)$ | $(-y + 3)(y + 2)$ |
| b) $(t + 1)(5t + 3)$ | $(-x + 10)(x + 3)$ | $(5x + 4)(x + 5)$ | $(y + 7)(2y + 6)$ |

| | | | |
|-----------------------------|--------------------------|--------------------------|----------------------------|
| c) $(3x + 7)(4x + 3)$ | $(-x + 8)(2x + 7)$ | $(4x + 7)(4x + 9)$ | $(-x + 6)(-x + 8)$ |
| d) $(6x + 5)(2x + 7)$ | $(-x + 8)(8x + 3)$ | $(3x + 9)(3x + 8)$ | $(12x + 12)(11x + 11)$ |
| e) $(-x - 3)(-x - 5)$ | $(3v - 5)(2v - 1)$ | $(7x - 9)(5x - 5)$ | $(3y - 3)(5y - 2)$ |
| f) $(2t - 1)(5t - 3)$ | $(3x - 10)(6x - 3)$ | $(2x - 4)(2x - 5)$ | $(5y - 7)(4y - 6)$ |
| g) $(6x - 7)(x - 3)$ | $(x - 8)(5x - 7)$ | $(4x - 7)(x - 9)$ | $(x - 6)(2x - 8)$ |
| h) $(-x - 5)(-x - 7)$ | $(-3x - 8)(5x - 3)$ | $(-5x - 9)(-5x - 8)$ | $(-4x - 12)(x - 11)$ |
| i) $(x + 3)(3x - 5)$ | $(v - 5)(7v + 1)$ | $(x + 9)(-9x - 5)$ | $(-2y - 3)(y + 2)$ |
| j) $(2t - 1)(2t + 3)$ | $(9x + 10)(8x - 3)$ | $(-2x - 4)(-x + 5)$ | $(5y + 7)(6y - 6)$ |
| k) $(7x - 7)(3x + 3)$ | $(-3x + 8)(-5x - 7)$ | $(3x - 7)(12x + 9)$ | $(-x - 6)(10x + 8)$ |
| l) $(2x + 5)(-3x - 7)$ | $(5x - 8)(5x + 3)$ | $(4x + 9)(-x - 8)$ | $(-6x - 12)(-2x + 11)$ |
| m) $(5r + 1)(5r + 1)$ | $(7x + 10)(7x + 10)$ | $(2x + 7)(2x + 7)$ | $(3y + 8)(3y + 8)$ |
| n) $(-2x + 5)(-2x + 5)$ | $(-z + 2)(-z + 2)$ | $(6m + 6)(6m + 6)$ | $(-5x + 4)(-5x + 4)$ |
| o) $(7y + 11)(7y + 11)$ | $(-5x + 12)(-5x + 12)$ | $(-4x + 3)(-4x + 3)$ | $(10x + 9)(10x + 9)$ |
| p) $(3r - 1)(3r - 1)$ | $(5x - 10)(5x - 10)$ | $(3x - 7)(3x - 7)$ | $(5y - 8)(5y - 8)$ |
| q) $(-4x - 5)(-4x - 5)$ | $(7z - 2)(7z - 2)$ | $(-3m - 6)(-3m - 6)$ | $(-x - 4)(-x - 4)$ |
| r) $(8y^2 - 11)(8y^2 - 11)$ | $(2x^2 - 12)(2x^2 - 12)$ | $(8x^3 - 3)(8x^3 - 3)$ | $(5x^2 - 9)(5x^2 - 9)$ |
| s) $(5r - 1)(5r + 1)$ | $(4x - 10)(4x + 10)$ | $(2x + 7)(2x - 7)$ | $(5y - 8)(5y + 8)$ |
| t) $(-x + 5)(-x - 5)$ | $(3z + 2)(3z - 2)$ | $(8m + 6)(8m - 6)$ | $(5x^2 + 4)(5x^2 - 4)$ |
| u) $(2x^3 + 5)(5x^4 + 3)$ | $(3y^2 - 5)(7y^3 - 10)$ | $(y^2 + x^2)(y^2 - x^2)$ | $(3xy^2 + 5y)(2x^2y + 3x)$ |