6-2 Factoring

In math most everything has an opposite. Multiplication's opposite is division. Addition's opposite is subtraction. Distribution and FOIL are two kinds of multiplication. The opposite is factorization.

Review distribution and factoring from previous lessons.
$4x^2 - 20x$ factors to $4x(x - 5)$ $\quad 7y(3y + 8) = 21y^2 + 56y$
Always see if you can "pull out" a common factor from each term in a trinomial.

Using a Pattern
One way to factor a trinomial is to notice a pattern. FOIL this $(x - 5)(x + 5)$. Did you know the answer without actually foiling because of the homework done before?

Following the pattern, what did $x^2 - 9$ look like before the foiling was done? $(x - 3)(x + 3)$
$(x + 3)(x - 3)$ is the same thing because of the commutative property of multiplication.

Practice: Use the pattern $a^2 - b^2 = (a - b)(a + b)$
a) $x^2 - 25$ $\quad x^2 - 49$ $\quad x^2 - 81$ $\quad x^2 - 100$

Another pattern you may have noticed in the FOIL homework is $a^2 + 2ab + b^2 = (a + b)(a + b)$. This pattern is not as obvious as the first pattern. FOIL $(x+3)(x+3) = x^2 + 6x + 9$. The last term is a perfect square. The middle term is two times the root of the last term.

Example: $x^2 + 10x + 25$ 25 is a perfect square. 10 is twice 5 which is the root of 25
The factors are $(x + 5)(x + 5)$

Practice:
b) $x^2 + 16x + 64$ $\quad x^2 + 8x + 16$ $\quad x^2 + 14x + 49$ $\quad x^2 + 12x + 36$

$x^2 - 6x + 9$ is another similar pattern. The last term is a perfect square and the middle coefficient is negative the double of the root of that square. $x^2 - 6x + 9 = (x - 3)(x - 3)$

Practice:
c) $x^2 - 16x + 64$ $\quad x^2 - 8x + 16$ $\quad x^2 - 14x + 49$ $\quad x^2 - 12x + 36$

Practice: Try factoring the following using the appropriate pattern.
d) $x^2 - 100$ $\quad x^2 - 4x + 4$ $\quad x^2 - 9$ $\quad x^2 + 2x + 1$

e) $x^2 + 10x + 25$ $\quad x^2 - 36$ $\quad x^2 - 10x + 25$ $\quad x^2 - 16$

f) $x^2 - 1$ $\quad x^2 - 2x + 1$ $\quad x^2 + 4x + 4$ $\quad x^2 + 20x + 100$

g) $x^2 - 20x + 100$ $\quad x^2 + 18x + 81$ $\quad x^2 - 64$ $\quad x^2 - 18x + 81$

h) $5x^2 - 500$ $\quad 8x^2 - 32x + 32$ $\quad 7x^2 - 63$ $\quad 10x^2 + 20x + 10$

Factor out a common factor
i) $3x^2 + 30x + 75$ $\quad 5x^2 - 180$ $\quad 2x^2 - 20x + 50$ $\quad 3x^2 - 48$

j) $12x^2 - 12$ $\quad 2yx^2 - 4yx + 2y$ $\quad yx^2 + 4yx + 4y$ $\quad 7x^2 + 140x + 700$

k) $x^3 - 20x^2 + 100x$ $\quad x^3 + 18x^2 + 81x$ $\quad 5y^2x^2 - 320y^2$ $\quad yx^2 - 18yx + 81y$

Factoring Without a Pattern

x^2-7x+12 doesn't fit the patterns shown, but the expression can still be factored into (x-3)(x-4).

We do have an algorithm for finding the factors of a trinomial with a leading coefficient of 1. A leading coefficient is the constant in the term with the highest power.

Step 1: Factor out any common factors.
Step 2: List all factors of the last term.
Step 3: Find the pair of factors of the last term whose sum is the coefficient of the middle term.
Step 4: Write the factors of the trinomial.
Step 5: Check by FOIL.

Step through the example x^2-7x+12. This is already listed in the appropriate order.

Step 1: There are no common factors.
Step 2: The pairs of factors for +12 are listed.
Step 3: The pair that adds to -7(the middle coefficient) is -3 and -4.
Step 4: (x - 3)(x - 4) Note: (x - 4)(x - 3) is the same answer. Why?
Step 5: Check. (x - 3)(x - 4)= x^2- 4x - 3x +12 = x^2-7x+12

+12		Adds to -7
1	12	13
-1	-12	-13
2	6	8
-2	-6	-8
3	4	7
-3	-4	-7

Practice: Follow the given steps.
x^2-x-20
Step 1: Are there any common factors?
Step 2: List the pairs of factors of -20
Step 3: Which pair adds to -1?
Step 4: Fill in the blanks. (x)(x)
Step 5: Check with foil.

After working many problems, you will begin to see the factors intuitively without using the steps listed.

a) x^2 + 4x - 21 x^2 - 5x + 4 x^2 + 8x + 15 x^2 + 6x - 7

b) x^2 + 9x + 18 x^2 - 5x+6 x^2 - 2x - 8 x^2 -x - 6

c) x^2 +7x +10 x^2 +10x - 11 x^2 + 9x + 14 x^2 + 8x + 12

d) x^2 - 4x - 12 x^2 + 26x + 25 x^2 - 12x +32 x^2 - 9x + 20

e) $5x^2$ – 5x - 30 $8x^2$ - 40x + 32 $7x^2$ + 63x +126 $10x^2$ - 20x - 80
Factor out a common factor first.

f) $3x^2$ + 30x + 75 $5x^2$ - 180 $2x^2$ - 20x + 50 $3x^2$ + 30x + 48

g) $12x^2$ –84x+ 144 $2yx^2$ - 12yx + 10y yx^2 + 2yx + y $7x^2$ + 56x + 105

h) x^3 - $5x^2$ - 14x x^3 + $11x^2$ + 18x $5y^2x^2$ - $50y^2$x - $280y^2$ yx^2 - yx + 12y

This works for a leading coefficient of 1. Notice when you did FOIL problems this is not the only kind of problem to factor. In future math classes, you will learn how to factor trinomials with a leading coefficient other than one.